

Bridging the Gap: Year 12 to Year 13 Mathematics Diagnostic

Complete and submit this diagnostic test before your first summer lesson. It covers Year 1 A level pure maths topics that are developed further in Year 2, together with skills needed throughout both years. I will review your answers and working before we begin, so that our summer lessons build on what you already understand and focus on the topics you need to strengthen before starting Year 13.

Suggested time 110 minutes Total 90 marks

The suggested time of 110 minutes is a guide only. Take as long as you need and record your total time. You may use a calculator and a formula booklet. Attempt every question and show all stages of your working in the spaces provided. Give exact answers unless otherwise stated. Record any other help used, and submit your completed test before checking the answers.

Name _____ Date _____ Time taken _____

1 Given that $p = \log_a 9$ and $q = \log_a 25$, where $a > 0$ and $a \neq 1$, find in terms of p and/or q ,

(a) $\log_a 81$ (1)

(b) $\log_a 75$ (2)

(c) $\log_a 15 \times \log_a \left(\frac{3}{5}\right)$. (2)

2 The curves C_1 and C_2 have equations $y = 2e^x$ and $y = 3 - e^{0.5x}$ respectively.

Find the exact coordinates of the point of intersection of C_1 and C_2 . Show all stages of your working. (5)

3 The quadratic equation $kx^2 + 4x + k = 0$ has no real roots.

Determine the set of possible values of k . (3)

- 4 The binomial expansion of $(1 + ax)^8$, up to and including the term in x^2 , is $1 - 6x + kx^2$, where a and k are constants.

(a) Show that $a = -\frac{3}{4}$. (2)

(b) Hence find the value of k . (2)

(c) Using the expansion, estimate $(1.03)^8$, giving your answer to 4 decimal places. (2)

- 5 The curve C has equation $y = \frac{3}{x}$, where $x \neq 0$.

(a) Sketch C , stating the equations of its asymptotes. (2)

(b) The point $P = (1, 3)$ lies on C . The curve is transformed to $y = 2f(x - 4) + 1$, where $f(x) = \frac{3}{x}$. Find the coordinates of the image of P .

(2)

(c) State the equation of the horizontal asymptote of $y = 2f(x - 4) + 1$.

(1)

6 The polynomial $f(x) = 2x^3 - 3x^2 - 11x + 6$.

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$.

(2)

(b) Use polynomial division to express $f(x)$ in the form $(x - 3)q(x)$, where $q(x)$ is a quadratic polynomial.

(3)

(c) Hence solve $f(x) = 0$.

(2)

7 Consider the equation $2\log_2 x = \log_2(kx - 1) + 2$, where k is a positive constant.

(a) Show that the equation can be written as $x^2 - 4kx + 4 = 0$.

(3)

(b) Given that the original equation has exactly one real solution, find the value of k and the solution for x .

(3)

8 A hot drink cools in a room. Its temperature T degrees Celsius is modelled by $T = 20 + ae^{-kt}$, where t is the time in minutes and a and k are positive constants. Initially, the temperature is 90°C . After 5 minutes, it is 60°C .

(a) Write down the value of a and explain what the value 20 represents in this model.

(2)

(b) Find the value of k , giving your answer to 4 decimal places.

(2)

(c) Find the time taken for the drink to reach 35°C , giving your answer to 3 significant figures.

(3)

9 Show all stages of your working in this question.

(a) Show that $\sin \theta(5\sin \theta - 2\cos \theta) = 2$ can be written as $3\tan^2 \theta - 2\tan \theta - 2 = 0$. (3)

(b) Hence solve $\sin \theta(5\sin \theta - 2\cos \theta) = 2$ for $0^\circ \leq \theta < 360^\circ$, giving your answers to 1 decimal place. (4)

(c) Hence find the smallest positive solution of $\sin 2\alpha(5\sin 2\alpha - 2\cos 2\alpha) = 2$. (1)

10 A curve has equation $y = \frac{2}{3}x^3 - 3x^2 - 8x + 5$.

(a) Find $\frac{dy}{dx}$, writing your answer in simplest form. (2)

(b) Hence find the range of values of x for which y is decreasing. (4)

11 The curve C has equation $y = x^3 - 3x$. The point $P = (2,2)$ lies on C .

(a) Find the gradient of C at P . (2)

(b) Find an equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers. (3)

12 The gradient of a curve is given by $\frac{dy}{dx} = 3x^2 - 4x + \frac{2}{x^2}$, where $x > 0$. The curve passes through (1,4).

Find the equation of the curve, giving your answer in the form $y = f(x)$.

(5)

13 A finite region R is bounded by the curve $y = 3x^2 + 2$, the y axis and the line $y = 14$.

(a) Find the positive x coordinate of the point where the curve meets the line $y = 14$.

(1)

(b) Using integration, find the exact area of R . Show all stages of your working.

(4)

14 Find the following indefinite integrals, giving each answer in its simplest form.

(a) Type equation here.

$$\int (6x^2 - 4x + 5) dx$$

(3)

(b) Type equation here.

$$\int \frac{3x^2 + 4}{\sqrt{x}} dx$$

(4)

15 Relative to the origin O , the points A and B have position vectors $2\mathbf{i} - 3\mathbf{j}$ and $8\mathbf{i} + 5\mathbf{j}$ respectively.

(a) Find the vector AB and its magnitude.

(3)

(b) The point C lies on the line segment AB such that $AC:CB = 1:2$. Find the position vector of C .

(3)

16 Given that $\tan \theta = \frac{5}{12}$ and $180^\circ < \theta < 270^\circ$,

(a) find the exact values of $\sin \theta$ and $\cos \theta$,

(3)

(b) hence find the exact value of $\sin^2 \theta - \cos^2 \theta$.

(1)