

ESAT Formulae and Calculator Free Practice

In A-level Maths, you can use a calculator and formula booklet. Neither is available in the ESAT. This checklist covers formulas and calculations you might otherwise rely on those tools for. Each topic includes a reminder and a practice question.

Mathematics 1 Core

1 Quadratic formula

Recall $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$ with $a \neq 0$.

Solve $3x^2 - 7x - 6 = 0$ exactly.

2 Discriminant

Use $\Delta = b^2 - 4ac$: positive means two distinct real roots, zero means one repeated root, and negative means no real roots.

Find k such that $x^2 - 6x + k = 0$ has exactly one real root.

3 Simultaneous equations by elimination

Choose a multiple of one equation so a variable cancels on adding or subtracting.

Solve $2x + 3y = 11$ and $x - y = 3$ by elimination.

4 Simultaneous equations by substitution

Substitute one equation into the other, solve the resulting equation, then recover both coordinates.

Find every intersection of $y = x + 1$ and $y = x^2 - 3x + 1$.

5 Exact fraction arithmetic

Cancel common factors before multiplying fractions; use a common denominator only when adding.

Calculate $\frac{35}{48} \times \frac{32}{21} + \frac{5}{9}$ exactly.

6 Surd arithmetic

Extract square factors before collecting like surds.

Simplify $\sqrt{72} - 2\sqrt{8} + \sqrt{50}$.

7 Rationalising a surd denominator

Multiply by the conjugate when the denominator is a difference or sum of surds.

Rationalise $\frac{3}{\sqrt{5} - \sqrt{2}}$.

8 Arithmetic nth term

Recall $u_n = a + (n - 1)d$.

An arithmetic sequence begins at 7 with common difference 3. Find its 20th term.

9 Standard form and powers of ten

Write numbers as $a \times 10^n$ with $1 \leq a < 10$; multiply the number parts and add the powers.

- a) Write 4.7×10^{-3} as an ordinary decimal.
- b) Write 58 000 in standard form.
- c) Calculate $(3 \times 10^4)(2 \times 10^{-2})$.

10 SI prefixes

Know the prefix symbols: nano (n) = 10^{-9} ; micro (μ) = 10^{-6} ; milli (m) = 10^{-3} ; centi (c) = 10^{-2} ; deci (d) = 10^{-1} ; kilo (k) = 10^3 ; mega (M) = 10^6 ; giga (G) = 10^9 .

- a) Convert 7.5 mg to grams.
- b) Convert 0.004 km to metres.
- c) Convert 2 MW to watts.

11 Length area volume and speed conversions

For squared or cubed units, square or cube the conversion factor. Convert compound units in both the numerator and denominator.

- a) Convert 250 cm^2 to m^2 .
- b) Convert 3 L to cm^3 .
- c) Convert 36 km h^{-1} to m s^{-1} .

12 Two successive percentage changes

Apply each percentage change to the new value; for a 10% increase, multiply by 1.1.

A price of £80 increases by 10% and then by another 10%. Find the final price without a calculator.

13 Area of a parallelogram

Recall $A = bh$, where h is the perpendicular height.

Find the area of a parallelogram with base 7 cm and perpendicular height 5 cm.

14 Area of a trapezium

Recall $A = \frac{1}{2}(a + b)h$, where a and b are the parallel sides.

Find the area of a trapezium with parallel sides 6 cm and 10 cm and height 4 cm.

15 Area of a rhombus

Using diagonals of lengths d_1 and d_2 , recall $A = \frac{1}{2}d_1d_2$.

Find the area of a rhombus with diagonals 8 cm and 15 cm.

16 Circle circumference and area

Recall $C = 2\pi r$ and $A = \pi r^2$.

A circle has radius 3 cm. Find its circumference and area exactly.

17 Arc and sector in degrees

An angle of θ° is $\frac{\theta}{360}$ of a full turn; use this fraction of the circle's circumference or area.

A circle of radius 6 cm has a sector of angle 60° . Find its arc length and area exactly.

18 Cuboid volume

Recall $V = lwh$.

Find the volume of a $2 \times 3 \times 4$ cuboid.

19 Cuboid surface area

Recall total surface area $A = 2(lw + lh + wh)$.

Find the total surface area of a $2 \times 3 \times 4$ cuboid.

20 Prisms

Recall $V = \text{cross-sectional area} \times \text{length}$; include all faces for total surface area.

A triangular prism is 10 long. Its cross-section is a right triangle with sides 3,4,5. Find its volume.

21 Prism surface area

Add the areas of the two congruent ends and each side face.

A triangular prism is 10 long and has a right-triangle cross-section with sides 3,4,5. Find its total surface area.

22 Cylinder volume

Recall $V = \pi r^2 h$.

Find the volume of a cylinder with radius 3 and height 8.

23 Cylinder surface area

Recall total surface area $A = 2\pi rh + 2\pi r^2$ for a closed cylinder.

Find the total surface area of a closed cylinder with radius 3 and height 8.

24 Exact trig values

Know sin, cos and tan at $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$ and the matching radian angles.

Function	0°	30°	45°	60°	90°
sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0

Function	0°	30°	45°	60°	90°
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	undefined

Evaluate exactly $\sin 30^\circ + \cos 60^\circ + \tan 45^\circ$.

25 Quadrant signs and reference angles

Use the acute reference angle together with the quadrant signs.

Evaluate $\cos 120^\circ + \sin 150^\circ$ exactly.

26 Periodicity and solutions

Know the graph symmetry and periods of sine, cosine and tangent.

Solve $\sin x = \frac{1}{2}$ for $0 \leq x < 2\pi$.

Mathematics 2 Extension

27 Quadratic inequalities

Find the roots and use the sign or shape of the quadratic; check the endpoints.

Solve $2x^2 - 5x - 3 \leq 0$.

28 Factorable cubic equations

Try a simple rational root, factor out the corresponding linear expression, then solve the remaining quadratic.

Solve $2x^3 - 3x^2 - 8x + 12 = 0$ exactly.

29 Arithmetic sum

Recall $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$.

Find the sum of the first 20 terms of the sequence 7, 10, 13, ...

30 Geometric nth term

Recall $u_n = ar^{n-1}$.

Find the 7th term of 24, 12, 6, ...

31 Finite geometric sum

Recall $S_n = \frac{a(1-r^n)}{1-r}$ for $r \neq 1$.

Find the sum of the first 12 terms of 3, 6, 12, ...

32 Infinite geometric sum

Recall $S_\infty = \frac{a}{1-r}$ when $|r| < 1$.

Find the sum to infinity of $24 + 12 + 6 + \dots$.

33 Sigma notation for arithmetic sums

Read both index limits and use $\sum_{r=1}^n r = \frac{n(n+1)}{2}$ where helpful.

Evaluate $\sum_{r=1}^{12} (3r - 1)$.

34 Sigma notation for geometric sums

Expand a few terms to identify the first term and ratio.

Evaluate $\sum_{k=0}^6 2^k$.

35 Combinations without a calculator

Use $\binom{n}{r} = \frac{n!}{r!(n-r)!}$, cancelling before multiplying. Here $n! = n(n-1) \cdots 1$ and $0! = 1$.

Calculate without a calculator:

- a) $\binom{12}{2}$
- b) $\binom{10}{3}$
- c) $\binom{8}{4}$

36 Binomial coefficients

In $(a+b)^n$, the coefficient of $a^{n-r}b^r$ is $\binom{n}{r}$.

Find the coefficient of x^3 in $(x-2)^7$.

37 Exact logarithms

Recall that $\log_a b = c$ means $a^c = b$. For exact values, rewrite the argument as a power of the base.

Calculate without a calculator:

- a) $\log_2 32$;
- b) $\log_3 \left(\frac{1}{27}\right)$.

38 Sine rule

Recall $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$; be alert to the ambiguous case.

In a triangle, a side of length 6 is opposite 30° . Find the side opposite 45° .

39 Cosine rule

Recall $a^2 = b^2 + c^2 - 2bc \cos A$ for the angle A between sides b and c .

Two sides are 5 and 7, with included angle 60° . Find the third side exactly.

40 Triangle area

Recall $A = \frac{1}{2}ab \sin C$, where C is the included angle.

Find the area of a triangle with sides 8 and 6 enclosing 30° .

41 Degree and radian conversion

Recall $180^\circ = \pi$ radians.

Express 135° in radians.

42 Arc length

Use $s = r\theta$ with θ in radians.

Find the length of an arc with radius 6 and angle $\pi/3$.

43 Sector area

Use $A = \frac{1}{2}r^2\theta$ with θ in radians.

Find the area of a sector of radius 6 and angle $\pi/3$.

44 Segment area

Subtract the triangular area from the corresponding sector area using $A = \frac{1}{2}r^2(\theta - \sin\theta)$ with θ in radians.

Find the area of the minor segment in a circle of radius 6 with central angle $\pi/3$.

45 Pythagorean identity

Use $\sin^2x + \cos^2x = 1$ and the quadrant to choose the sign.

Given $\cos x = \frac{3}{5}$ and x is reflex, find $\sin x$.

46 Tangent identity

Recall $\tan x = \frac{\sin x}{\cos x}$ when $\cos x \neq 0$.

If $\sin x = \frac{3}{5}$ and $\cos x = -\frac{4}{5}$, find $\tan x$.

47 Transformed trig equations

Solve for the whole argument first, then find every solution in the required interval.

Solve $\sin(2x) = \frac{\sqrt{3}}{2}$ for $0 \leq x < 2\pi$.

48 Definite integrals

Find an antiderivative F , then use $\int_a^b f(x) dx = F(b) - F(a)$.

Evaluate $\int_1^3 (2x^2 - x) dx$ exactly.

49 Area between a curve and a line

Area enclosed below $f(x)$ and above $g(x)$ is $\int_a^b (f(x) - g(x)) dx$, where a and b are their intersection points.

Find the area enclosed by $y = x^2$ and $y = 2x$.

50 Reversing and joining integration limits

Use $\int_b^a f dx = -\int_a^b f dx$ and $\int_a^c f dx + \int_c^b f dx = \int_a^b f dx$.

Given $\int_0^2 f(x) dx = 3$ and $\int_2^5 f(x) dx = -1$, find $\int_5^0 f(x) dx$.

51 Trapezium rule

With $h = (b - a)/n$, use $\int_a^b f(x) dx \approx \frac{h}{2} [y_0 + 2(y_1 + \cdots + y_{n-1}) + y_n]$.

Use two equal strips to approximate $\int_0^2 x^2 dx$. State whether the result is above or below the exact value.

Answers for checking

No.	Answer	No.	Answer	No.	Answer
1	$x = 3, -\frac{2}{3}$.	18	24.	35	A. 66; B. 120; C. 70.
2	$k = 9$.	19	52.	36	560.
3	$(x, y) = (4, 1)$.	20	60.	37	A. 5; B. -3.
4	$(0, 1)$ and $(4, 5)$.	21	132.	38	$6\sqrt{2}$.
5	$\frac{5}{3}$.	22	72π .	39	$\sqrt{39}$.
6	$7\sqrt{2}$.	23	66π .	40	12.
7	$\sqrt{5} + \sqrt{2}$.	24	2.	41	$\frac{3\pi}{4}$.
8	64.	25	0.	42	2π .
9	A. 0.0047; B. 5.8×10^4 ; C. 600.	26	$x = \pi/6, 5\pi/6$.	43	6π .
10	A. 0.0075 g; B. 4 m; C. 2×10^6 W.	27	$-\frac{1}{2} \leq x \leq 3$.	44	$6\pi - 9\sqrt{3}$.
11	A. 0.025 m^2 ; B. 3000 cm^3 ; C. 10 m s^{-1} .	28	$x = -2, \frac{3}{2}, 2$.	45	$-\frac{4}{5}$.
12	£96.80.	29	710.	46	$-\frac{3}{4}$.
13	35 cm^2 .	30	$\frac{3}{8}$.	47	$x = \pi/6, \pi/3, 7\pi/6, 4\pi/3$.
14	32 cm^2 .	31	12285.	48	$\frac{40}{3}$.
15	60 cm^2 .	32	48.	49	$\frac{4}{3}$.
16	$6\pi \text{ cm}$ and $9\pi \text{ cm}^2$.	33	222.	50	-2.
17	$2\pi \text{ cm}$ and $6\pi \text{ cm}^2$.	34	127.	51	3; overestimate (exact value $8/3$).

Scope

Based on the UAT-UK ESAT Content Specification for the Mathematics 1 and Mathematics 2 modules. Science-module formulas are not included. Cone and sphere formulas are supplied if required by the specification.