

Question 1:

6.

$$a = \log_2 x$$

$$b = \log_2(x + 8)$$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$

(1)

(b) $\log_2(x^2 + 8x)$

(2)

(c) $\log_2\left(8 + \frac{64}{x}\right)$

Give your answer in simplest form.

(3)

(a) $\log_2 \sqrt{x} = \log_2 x^{1/2}$

$$= \frac{1}{2} \log_2 x$$

$$= \underline{\underline{\frac{1}{2}a}}$$

(b) $\log_2(x^2 + 8x) = \log_2(x(x + 8))$

$$= \log_2 x + \log_2(x + 8)$$

$$= \underline{\underline{a + b}}$$

(c) $\log_2\left(8 + \frac{64}{x}\right) = \log_2\left(\frac{8(x + 8)}{x}\right)$

$$= \log_2 8 + \log_2(x + 8) - \log_2 x$$

$$= \underline{\underline{3 + b - a}}$$

Question 2:

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)

$$\log_3 \left(\frac{12y + 5}{1 - 3y} \right) = 2$$

$$\frac{12y + 5}{1 - 3y} = 3^2$$

$$12y + 5 = 9(1 - 3y)$$

$$12y + 5 = 9 - 27y$$

$$39y = 4$$

$$\underline{y = \frac{4}{39}}$$

Question 3:

1. (a) Given that

$$2\log(3-x) - \log(21-2x) = 0$$

show that

$$x^2 - 4x - 12 = 0$$

(3)

- (b) (i) Write down the roots of $x^2 - 4x - 12 = 0$

- (ii) Which of these two roots is **not** a solution of

$$2\log(3-x) - \log(21-2x) = 0$$

giving a reason for your answer.

(2)

$$(a) \quad \log(3-x)^2 - \log(21-2x) = 0$$

$$\log \frac{(3-x)^2}{21-2x} = 0$$

$$\frac{(3-x)^2}{21-2x} = 1$$

$$(3-x)^2 = 21-2x$$

$$9-6x+x^2 = 21-2x$$

$$\underline{x^2 - 4x - 12 = 0}$$

$$(b) (i) \quad (x-6)(x+2) = 0$$

$$\underline{x=6}, \quad \underline{x=-2}$$

$$(ii) \quad \underline{x=6} \text{ is } \underline{\text{not}} \text{ a solution since } 3-x = 3-6 = \underline{-3 < 0}$$

Question 4:

12.

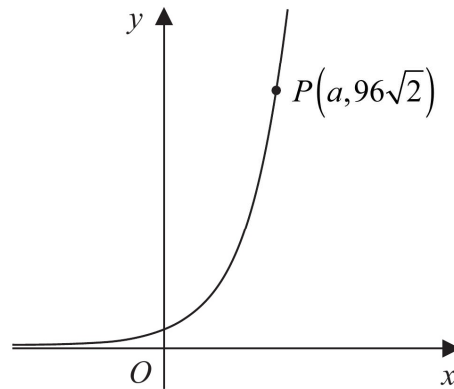


Figure 6

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 6 shows a sketch of part of the curve with equation

$$y = 3 \times 2^{2x}$$

The point $P(a, 96\sqrt{2})$ lies on the curve.

(a) Find the exact value of a .

(3)

The curve with equation $y = 3 \times 2^{2x}$ meets the curve with equation $y = 6^{3-x}$ at the point Q .

(b) Show that the x coordinate of Q is

$$\frac{3 + 2 \log_2 3}{3 + \log_2 3}$$

$$(a) \quad x = a, \quad y = 96\sqrt{2}$$

$$96\sqrt{2} = 3 \times 2^{2a}$$

$$32 \times 2^{1/2} = 2^{2a}$$

$$2^5 \times 2^{1/2} = 2^{2a}$$

$$2^{11/2} = 2^{2a}$$

$$2a = \frac{11}{2}$$

$$a = \frac{11}{4}$$

$$(b) \quad 3 \times 2^{2x} = 6^{3-x} \quad (5)$$

$$3 \times 2^{2x} = 2^{3-x} \times 3^{3-x}$$

$$2^{3x-3} = 3^{2-x}$$

$$\log_2 2^{3x-3} = \log_2 3^{2-x}$$

$$3x-3 = (2-x)\log_2 3$$

$$3x + x \log_2 3 = 3 + 2 \log_2 3$$

$$x = \frac{3 + 2 \log_2 3}{3 + \log_2 3}$$

Question 5:

4. Using the rules of logarithms, solve the equation

$$\log_2(x+2) + \log_2(x+3) - 2\log_2 x = 1$$

(4)

$$\log_2 \frac{(x+2)(x+3)}{x^2} = 1$$

$$\frac{x^2 + 5x + 6}{x^2} = 2$$

$$x^2 + 5x + 6 = 2x^2$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$\therefore \underline{x = 6}$$

Question 6:

3.

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 3x^3 - 20x^2 + 45x - 22$$

(a) Use the factor theorem to show that $(3x - 2)$ is a factor of $f(x)$

(2)

(b) Hence show that the equation $f(x) = 0$ has only one real root.

(4)

(c) Hence find the exact value of x for which

$$3(\ln 2x)^3 - 20(\ln 2x)^2 + 45 \ln 2x - 22 = 0$$

$$(a) \quad f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^3 - 20\left(\frac{2}{3}\right)^2 + 45\left(\frac{2}{3}\right) - 22$$

(2)

$$= \frac{8}{9} - \frac{80}{9} + \frac{90}{3} - 22 = 0$$

$$\therefore 3x - 2 \text{ is a factor of } f(x)$$

$$(b) \quad f(x) = (3x - 2)(ax^2 + bx + c)$$

$$\text{Compare } x^3 : 3 = 3a \Rightarrow a = 1$$

$$1 : -22 = -2c \Rightarrow c = 11$$

$$x : 45 = 3c - 2b \Rightarrow b = -6$$

$$\therefore f(x) = (3x - 2)(x^2 - 6x + 11)$$

$$b^2 - 4ac = (-6)^2 - 4 \times 1 \times 11 = -8 < 0$$

$$\therefore x^2 - 6x + 11 \neq 0$$

$$\text{so } \underline{x = \frac{2}{3}} \text{ is the } \underline{\text{only}} \text{ root}$$

$$(c) \quad \ln 2x = \frac{2}{3}$$

$$2x = e^{\frac{2}{3}}$$

$$\underline{x = \frac{1}{2} e^{\frac{2}{3}}}$$