

TMUA & ESAT Mathematics Pre-Course Diagnostic

Prerequisite mathematics for TMUA and ESAT

Suggested time: Part A 30 minutes; Part B 25 minutes

ESAT Mathematics 1 only: complete Part A.

ESAT Mathematics 1 and Mathematics 2: complete Parts A and B.

TMUA: complete Parts A and B.

Name: _____ Date: _____

Instructions

- Do not use a calculator, notes or a formula sheet.
- Give exact answers where appropriate.
- Show sufficient working for your method to be identified.
- You may write your answers on this paper or on a separate sheet.
- Work through the questions in order. If a question is taking too long, leave it and return later.
- At the end of the time allowed for each part, record the last question you reached. You may then complete any unfinished questions without a time limit.
- This is a prerequisite diagnostic. It is designed to identify gaps in underlying knowledge and non-calculator fluency before exam-specific preparation begins, rather than to predict a TMUA or ESAT score.

Part A: Core prerequisite mathematics - 25 questions, 30 minutes

1. Evaluate

$$\frac{3}{4} + \frac{5}{6}$$

2. Calculate 84×25 without a calculator.

3. Find 15% of 240.

4. A price is increased by 20% and then reduced by 20%. By what percentage is the final price different from the original price?

5. The ratio $A:B$ is 5:2 and the ratio $B:C$ is 3:7. Find $A:C$ in its simplest form.

6. y is directly proportional to x^2 . When $x = 3$, $y = 18$. Find y when $x = 5$.

7. x is inversely proportional to $\sqrt{y}$. When $x = 6$, $y = 16$. Find y when $x = 8$.

8. Simplify fully

$$\frac{(6a^3b^{-2})(2a^{-1}b^5)}{3ab}.$$

9. Evaluate exactly

$$16^{-\frac{3}{4}}.$$

10. Simplify

$$\sqrt{12} + \sqrt{27}.$$

11. Simplify fully

$$\frac{x^2 - 9}{x^2 + x - 6}$$

and state any values of x for which the original expression is undefined.

12. Factorise completely

$$2x^2 - 7x - 15.$$

13. Make q the subject of

$$p = \frac{3q - r}{2q + r}.$$

14. Solve

$$\frac{2x - 1}{3} - \frac{x + 2}{4} = 1.$$

15. Solve simultaneously

$$\begin{aligned}3x + 2y &= 16, \\ 2x - y &= 1.\end{aligned}$$

16. Solve exactly

$$2x^2 - 5x - 3 = 0.$$

17. Find the complete set of values of x satisfying

$$(x - 4)(x + 2) < 0.$$

18. Write

$$x^2 - 6x + 11$$

in the form $(x - a)^2 + b$.

19. The line through $(2, 5)$ and $(8, k)$ is perpendicular to the line

$$2x - 3y + 7 = 0.$$

Find k .

20. The points $A(-2,3)$ and $B(6,-3)$ are joined. Find:

- (a) the midpoint of AB ;
- (b) the exact length of AB .

21. A right-angled triangle has perpendicular sides of length 6 cm and 8 cm. A similar triangle has hypotenuse 25 cm. Find the lengths of its two perpendicular sides.

22. Give the exact values of:

- (a) $\sin 30^\circ$;
- (b) $\cos 45^\circ$;

23. In a right-angled triangle,

$$\tan \theta = \frac{3}{4}, \quad 0^\circ < \theta < 90^\circ.$$

Find $\sin \theta$ and $\cos \theta$ exactly.

24. A regular polygon has an exterior angle of 24° . How many sides does it have?

25. Expand and simplify

$$(2 + x)^4$$

Part B: Additional prerequisite mathematics - 20 questions, 25 minutes

Complete Part B if you are taking TMUA or ESAT Mathematics 2.

26. An arithmetic sequence has first term 7 and common difference 3. Find the sum of the first 15 terms.

27. A geometric series has first term 81 and common ratio $\frac{1}{3}$. Find the sum to infinity.

28. A geometric sequence has first term 3 and common ratio 2. Find its 8th term.

29. A geometric series begins 3, 6, 12, ... Find the sum of its first 8 terms.

30. Find the coefficient of x^3 in the expansion of $(1 + 2x)^5$.

31. The function f is defined by

$$f(x) = 2x - 3.$$

Find $f^{-1}(x)$.

32. The functions f and g are defined by $f(x) = 2x - 1$ and $g(x) = x^2 + 3$. Find $f(g(x))$ and $g(f(x))$.

33. The graph of $y = f(x)$ is transformed to the graph of
 $y = 2f(x - 3) + 1$.
Describe the three transformations involved.

34. Solve

$$\log_2(x - 1) + \log_2(x + 1) = 3.$$

35. Solve

$$3^{2x-1} = 27.$$

36. Find the remainder when

$$2x^3 - 3x^2 + 4x - 5$$

is divided by $x - 2$.

37. The curve

$$y = x^2 + kx + 4$$

is tangent to the x -axis. Find all possible values of k .

38. Express 150° in radians, giving your answer in terms of π .

39. Simplify

$$\frac{1 - \cos^2 x}{\sin x},$$

for values of x for which the expression is defined.

40. Solve, for $0^\circ \leq \theta < 360^\circ$,

$$2\sin^2\theta - 3\sin\theta + 1 = 0.$$

41. Given

$$\sin\theta = \frac{3}{5}, \quad 90^\circ < \theta < 180^\circ.$$

Find $\cos\theta$ exactly.

42. The curve has equation

$$y = x^3 - 6x^2 + 9x.$$

Find the x -coordinates of its stationary points.

43. The curve has equation

$$y = x^2 + \frac{1}{x}.$$

Find the gradient of the tangent to the curve at $x = 1$.

44. Find

$$\int (6x^2 - 4x + 3) \, dx.$$

45. Find the area enclosed by the curves

$$\begin{aligned} y &= x^2 \\ y &= x. \end{aligned}$$

End of paper

Last question reached in Part A: _____ Part B: _____