

TMUA: Necessary and Sufficient Conditions

Necessary means it must be there for the conclusion to happen.

Sufficient means it is enough on its own to guarantee the conclusion.

Let A be the condition and B the conclusion.

To test necessity: must A be true for B to be true? If yes, A is necessary for B.

To test sufficiency: if A is true, must B be true? If yes, A is sufficient for B.

Case 1:

Consider positive integers

A: A number is a multiple of 3

B: A number is even

Necessary: No (2 is even (B is true) but not a multiple of 3 (A is false))

Sufficient: No (3 is a multiple of 3 (A is true) but not even (B is false))

A is not necessary and not sufficient for B

Case 2:

Consider positive integers

A: A number is divisible by 2

B: A number is a multiple of 4

Necessary: Yes (all multiples of 4 are divisible by 2, so if B is true, A is true)

Sufficient: No (6 is divisible by 2 (A is true) but is not a multiple of 4 (B is false))

A is necessary but not sufficient for B

Case 3:

Consider positive integers

A: A number is a multiple of 4

B: A number is divisible by 2

Necessary: No (6 is divisible by 2 (B is true) but is not a multiple of 4 (A is false))

Sufficient: Yes (all multiples of 4 are divisible by 2, so if A is true, B is true)

A is not necessary but is sufficient for B

Case 4:

Consider positive integers

A: A number is even

B: A number is divisible by 2

Necessary: Yes (all numbers divisible by 2 are even, so if B is true, A is true)

Sufficient: Yes (all even numbers are divisible by 2, so if A is true, B is true)

A is necessary and sufficient for B

Practice questions

Selected TMUA Paper 2 questions

2020 Paper 2, Question 6

6 The function $f(x)$ is defined for all real values of x .

Which of the following conditions on $f(x)$ is/are **necessary** to ensure that

$$\int_{-5}^0 f(x) \, dx = \int_0^5 f(x) \, dx$$

Condition I: $f(x) = f(-x)$ for $-5 \leq x \leq 5$

Condition II: $f(x) = c$ for $-5 \leq x \leq 5$, where c is a constant

Condition III: $f(x) = -f(-x)$ for $-5 \leq x \leq 5$

- A none of them
- B I only
- C II only
- D III only
- E I and II only
- F I and III only
- G II and III only
- H I, II and III

2020 Paper 2, Question 7

7 Consider the following conditions on a **parallelogram** $PQRS$, labelled anticlockwise:

- I length of PQ = length of QR
- II The diagonal PR intersects the diagonal QS at right angles
- III $\angle PQR = \angle QRS$

Which of these conditions is/are individually **sufficient** for the parallelogram $PQRS$ to be a square?

	Condition I is sufficient	Condition II is sufficient	Condition III is sufficient
A	yes	yes	yes
B	yes	yes	no
C	yes	no	yes
D	yes	no	no
E	no	yes	yes
F	no	yes	no
G	no	no	yes
H	no	no	no

2019 Paper 2, Question 5

5 Consider the following statement about the positive integers a , b and n :

($*$): ab is divisible by n

The condition 'either a or b is divisible by n ' is:

- A necessary but **not sufficient** for ($*$)
- B sufficient but **not necessary** for ($*$)
- C necessary and **sufficient** for ($*$)
- D **not necessary** and **not sufficient** for ($*$)

2019 Paper 2, Question 10

10 $PQRS$ is a quadrilateral, labelled anticlockwise.

Which one of the following is a **necessary** but **not** **sufficient** condition for $PQRS$ to be a parallelogram?

- A $PQ = SR$ and PS is parallel to QR
- B $PQ = SR$ and PQ is parallel to SR
- C $PQ = QR = SR = PS$
- D $PR = QS$

2021 Paper 2, Question 6

6 Consider the following two statements about the polynomial $f(x)$:

P : $f(x) = 0$ for exactly three real values of x

Q : $f'(x) = 0$ for exactly two real values of x

Which one of the following is correct?

- A P is **necessary** but **not sufficient** for Q .
- B P is **sufficient** but **not necessary** for Q .
- C P is **necessary and sufficient** for Q .
- D P is **not necessary** and **not sufficient** for Q .

2021 Paper 2, Question 8

8 Consider the following statement about the polynomial $p(x)$, where a and b are real numbers with $a < b$:

(*) There exists a number c with $a < c < b$ such that $p'(c) = 0$.

Which one of the following is true?

- A The condition $p(a) = p(b)$ is **necessary and sufficient** for (*)
- B The condition $p(a) = p(b)$ is **necessary** but **not sufficient** for (*)
- C The condition $p(a) = p(b)$ is **sufficient** but **not necessary** for (*)
- D The condition $p(a) = p(b)$ is **not necessary** and **not sufficient** for (*)

2021 Paper 2, Question 15

15 A circle has equation

$$x^2 + ax + y^2 + by + c = 0$$

where a , b and c are non-zero real constants.

Which one of the following is a **necessary and sufficient** condition for the circle to be tangent to the y -axis?

A $a^2 = 4c$

B $b^2 = 4c$

C $\frac{a}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

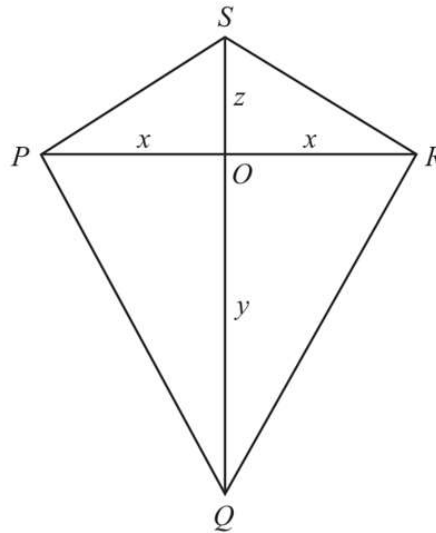
D $\frac{b}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

E $-\frac{a}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

F $-\frac{b}{2} = \sqrt{\frac{a^2 + b^2}{4} - c}$

2022 Paper 2, Question 11

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The diagram shows a kite $PQRS$ whose diagonals meet at O .

$$\begin{aligned}
 OP &= x \\
 OQ &= y \\
 OR &= x \\
 OS &= z
 \end{aligned}$$

Which of the following is **necessary and sufficient** for angle SPQ to be a right angle?

- A $x = y = z$
- B $2x = y + z$
- C $x^2 = yz$
- D $y = z$
- E $y^2 = x^2 + z^2$